8. Summary

In Chapter 2 of the dissertation we study Kolmogorov and Marcinkiewicz—Zygmund type strong laws of large numbers (SLLN's). Here Kolmogorov's SLLN is proved for pairwise independent weakly mean dominated random variables with multidimensional indices. Petrov showed in 1987 that the Marcinkiewicz SLLN holds for identically distributed random variables with arbitrary dependence structure, if $0 < r < 1$. We prove it for non-independent, weakly mean dominated random fields ($0 < r < 1$). Moreover we give a proof of Spitzer's theorem with similar assumptions.

There is an approach to prove the SLLN which uses directly a maximal inequality for normed sums. Inequalities of this kind are said to be of Hájek—Rényi type. They are not easy to obtain, but after the proof of the SLLN becomes an obvious problem. Fazekas and Klesov showed in 2000 that a Hájek—Rényi type inequality is a consequence of an appropriate maximal inequality for cumulative sums and the latter automatically implies the SLLN for sequences of random variables. In these results it is important that there are no restrictions on the dependence structure of random variables. In Chapter 3 we generalize these theorems for random fields. Several examples of applications are given in this chapter as well:

— An SLLN for logarithmically weighted sums. (We remark that such kind of SLLN's can be useful to prove almost sure central limit theorems.)
— A Marcinkiewicz—Zygmund type SLLN for random fields with superadditive moment structure.
— Brunk—Prohorov type theorems.

In Chapter 4 we study convergence rates in the laws of large numbers for general arrays of Banach space valued random elements. Some of our results are new for real variables, too. The main result is a generalization of a theorem of Jain, wich was published in 1975. The idea of the proof of the main theorem is the following. When we apply Hoffmann—Jørgensen's inequality, we use two different functions to obtain upper bounds for the two terms in the inequality. This theorem seems to be difficult, but when we choose appropriate weight functions we can obtain several known theorems for general arrays. We specialize our result for Banach spaces with some geometric property. Then we obtain new proofs for some results of Fazekas (1992) and Hu, Rosalsky, Szynal and Volodin (1999).

In Chapter 5 we study almost sure central limit theorems for random fields. In this topic Fazekas and Rychlik proved a general theorem in 2002. In

this chapter applying this result we prove an almost sure central limit theorem in so-called m-dependent case. For this reason we need a central limit theorem for m-dependent random fields, which was published by Prakasa Rao in 1981.

In Chapter 6 we give a version of Rosenthal's inequality for α-mixing fields. Rosenthal's inequalities are important tools to prove consistency of some estimators for weakly dependent random processes and fields. The first version of such inequalities was proved by Rosenthal in 1970 for independent random variables. Rosenthal's inequalities for mixing sequences were presented by Utev in 1984 and for mixing fields by Doukhan in 1994. However, Doukhan remarked that the proof of the interpolation lemma of Utev is "not clear". So the extension of Rosenthal's inequality from positive even integer exponents to arbitrary positive real exponents is an open problem. On the other hand, Doukhan presented Rosenthal's inequalities for α-mixing and for φ-mixing fields. Unfortunately, there is a gap in the proof of Doukhan. We want to summarize what is clear in the above mentioned proofs. Detailed proofs are given in the α-mixing case. The results and proofs of this chapter are slight modifications of the ones in Doukhan and Utev, however assumptions here are stronger than those in Doukhan's theorem.